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Penetration of the interplanetary magnetic field B_y into Earth's plasma sheet

L.-N. Hau

Institute of Space Science, National Central University, Chung-Li, Taiwan

G. M. Erickson

Center for Space Physics, Boston University, Boston, Massachusetts

Abstract. There has been considerable recent interest in the relationship between the cross-tail magnetic field component B_y and tail dynamics. The purpose of this paper is to give an overall description of the penetration of the interplanetary magnetic field (IMF) B_y into the near-Earth plasma sheet. We show that plasma sheet B_y may be generated by the differential shear motion of field lines and enhanced by the flux tube compression. The latter mechanism leads to a B_y analogue of the pressure-balance inconsistency (Erickson and Wolf, 1980) as flux tubes move from the far tail toward the Earth. The growth of B_y , however, may be limited by the dawn-dusk asymmetry in the shear velocity as a result of plasma sheet tilting. B_y penetration into the plasma sheet implies field-aligned currents flowing between hemispheres. These currents together with the IMF B_y related mantle field-aligned currents effectively shield the lobe from the IMF B_y .

1. Introduction

There is abundant observational evidence supporting the open model of Earth's magnetosphere. One indication is a high correlation between the interplanetary magnetic field (IMF) B_y and the observed cross-tail, magnetic field component B_y in the plasma sheet [e.g., *Lui*, 1984]. This result, as suggested by *Moses et al.* [1985], may be interpreted in terms of high-latitude convection patterns observed during periods of large IMF B_y , which show opposite senses of rotational flow v_y in conjugate hemispheres (v_y has the same sign as the IMF B_y in the northern hemisphere, as to be discussed in the next section). The asymmetric flow appearing on closed field lines causes an azimuthal tilt of nightside plasma sheet field lines, resulting in a B_y with the same direction as the IMF B_y . While this conceptually simple model suggested by *Moses et al.* [1985] may be used to interpret the observational feature of a larger B_y in the plasma sheet than the tail lobes [Fairfield, 1979; *Lui*, 1984; *Kaymaz et al.*, 1994a, b], it does not directly address the questions of how B_y enters the near-Earth plasma sheet [e.g., *McComas et al.*, 1986; *Nagai*, 1987] from the far tail [e.g., *Tsurutani et al.*, 1984; *Sibeck et al.*, 1985] and of why the penetration is stronger in the near tail than in the far tail [e.g., *Sergeev*, 1987]. As reported by *McComas et al.* [1986], there is an approximate hour correlation between the IMF B_y and the y component of the magnetic field in the plasma sheet. This delay is apparently due to the returning sunward convection in the tail, where the IMF-associated B_y field appearing in the open tail lobes may enter the closed field lines following the reconnection process [e.g., *Cowley*, 1981]. The purpose of this paper is to provide an overall description of how B_y may be carried into the near-Earth plasma sheet via the convection process. We show that plasma sheet B_y may be generated not only by the shear flow associated with the IMF B_y as described

by *Moses et al.* [1985] but also by flux tube compression during earthward convection. The latter mechanism leads to a B_y analogue of the pressure-balance inconsistency (PBI) discussed by *Erickson and Wolf* [1980]. The PBI argument states that standard quiet time magnetic field models are inconsistent with the notion of steady state adiabatic lossless plasma sheet convection. The difficulty is primarily due to much shorter near-Earth field line lengths than those in the distant plasma sheet. This causes near-Earth flux tubes to have much smaller pV^γ values than those in the far tail due to much smaller flux tube volume $V = \int ds/B$ there. Both the original PBI argument and existing modeling results [e.g., *Hau*, 1991; *Erickson*, 1992] assume $B_y = 0$. Since the presence of B_y leads to relatively longer field lines, one may speculate whether its presence can serve as a path to remove the PBI. Such a possibility will be examined in this paper. Our motivation is due to recent interest in the B_y effect on tail dynamics. As reported by *Sergeev et al.* [1993], a significant increase in B_y is occasionally observed in the near-Earth plasma sheet prior to the onset of a substorm. *Hau and Voigt* [1992] pointed out that within the context of two-dimensional MHD equilibrium theory, a sudden enhancement of B_y in the plasma sheet can act as an additional pressure that is able to reduce the equatorial normal magnetic field component and thus lead to the current disruption in a highly stretched magnetic field configuration established by adiabatic sunward convection [Hau, 1991; Erickson, 1992].

2. Dynamic Evolution of B_y

In this section we give a general explanation for the enhancement of B_y in the plasma sheet based on Faraday's law and the frozen-in-flux condition

$$\partial \mathbf{B} / \partial t = -\nabla \times \mathbf{E} \quad (1)$$

$$\mathbf{E} + \mathbf{v} \times \mathbf{B} = 0 \quad (2)$$

By use of $\nabla \cdot \mathbf{B} = 0$ and the continuity equation,

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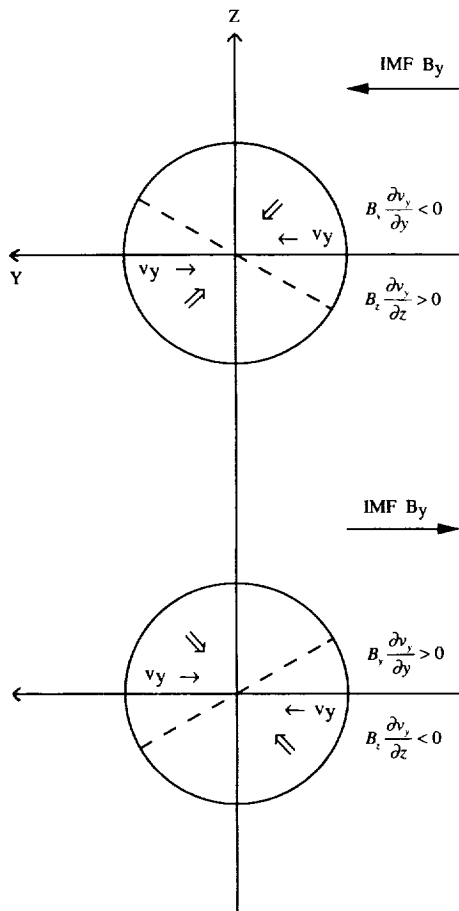


Figure 1. Magnetotail cross sections showing the tilting of the plasma sheet for positive (dawn to dusk) and negative (dusk to dawn) interplanetary magnetic field (IMF) B_y . The plasmas converge toward the neutral sheet, as indicated by the thick arrows, resulting in an asymmetric shear flow at the equatorial plane. The signs of the shear flow terms in (5) are also shown.

$$(\partial \rho / \partial t) + \nabla \cdot (\rho \mathbf{v}) = 0 \quad (3)$$

the time evolution of the y component magnetic field vector may be written as

$$\frac{dB_y}{dt} = \frac{B_y}{\rho} \frac{d\rho}{dt} + (\mathbf{B} \cdot \nabla) v_y \quad (4)$$

although the same equation may also be derived for B_x and B_z . Note that the coordinate system to be used throughout the paper is GSM coordinates such that positive x , y , and z are directed toward the Sun, the dusk, and north side of the Earth. Equation (4) clearly indicates that the change of B_y may be caused by compressing or expanding the plasma and by the differential shear motion as well. We discuss these two mechanisms separately.

For incompressible plasmas, i.e., $\rho = \text{const}$, the time evolution of B_y is entirely attributed to the shear flow term

$$\frac{dB_y}{dt} = \left(B_x \frac{\partial}{\partial x} + B_y \frac{\partial}{\partial y} + B_z \frac{\partial}{\partial z} \right) v_y \quad (5)$$

In the center of the plasma sheet where $B_x = 0$ (defining the so-called "neutral" sheet), the first term on the right-hand side

of (5) thus vanishes. If initially B_y is absent, then the last term in (5) is responsible for the generation of B_y in the plasma sheet. In particular, for positive B_z , as in Earth's quiet magnetosphere, an antisymmetric shear flow in opposite hemispheres may give rise to a neutral sheet B_y in the same sense as v_y in the northern hemisphere. The antisymmetric feature of v_y has been observed in high-latitude convection patterns [e.g., Moses *et al.*, 1985, and references therein] and in the magnetotail as well [McComas *et al.*, 1986] during the periods of large IMF B_y . The sense of v_y in correlation with the IMF B_y may be understood as follows. An IMF B_y is associated with a north-south electric field component E_z in the solar wind which may enter the closed field lines of the distant magnetotail via reconnection process and implies a rotational flow $v_y = E_z/B_x$ in the tail lobes. The north-south antisymmetric shear flow associated with the IMF B_y thus gives rise to the nightside plasma sheet B_y in the same direction as the IMF B_y . This mechanism is indeed the qualitative model proposed by Moses *et al.* [1985] for the generation of B_y in the plasma sheet. Since B_x changes sign in the dayside, there the plasma sheet B_y produced by the last term of (5) is in the opposite direction as the IMF B_y . Note also that the term is proportional to B_z . As shown by Fairfield [1986], the plasma sheet B_z is relatively larger at the flanks than in the center. Plasma sheet B_y resulting from the last term in (5) is thus larger at the flanks than in the center, a result being in agreement with the recent observations by Kaymaz *et al.* [1994b], who show that IMF B_y penetration can be as large as 35% at the flanks compared to 26% in the central portion of the plasma sheet.

The second term in (5) is due to the dawn-dusk asymmetry in v_y . Both observations and theoretical works have indicated that the magnetosphere exhibits a dawn-dusk asymmetry in association with the IMF B_y . In particular, in their open magnetotail model, Siscoe and Sanchez [1987] have shown that the plasma sheet at the dawnside may be tilted below and above the equatorial plane, respectively, for positive (dawn to dusk) and negative (dusk to dawn) IMF B_y . The three-dimensional magnetotail equilibrium models constructed by Bim [1990] clearly indicate the effect of plasma sheet tilting in the presence of B_y . Experimental evidence for the tilting of the plasma sheet induced by the IMF B_y is provided, for example, by Tsyganenko [1990], who found that the current sheet is progressively twisted down the tail as predicted by Cowley [1981]. Kaymaz *et al.* [1994b] further showed that the current sheet at $x = -33 R_E$ is tilted by $8.4^\circ \pm 1.7^\circ$ to $4.4^\circ \pm 1.3^\circ$, on average, for positive and negative IMF B_y , respectively. The tilting of the plasma sheet, in the simplest way, can be explained in terms of the simple reconnection model of the open magnetosphere [e.g., Stern, 1973; Cowley, 1981]. As shown in Figure 1, the flow converges toward the tilted plasma sheet (reconnection line), as indicated by the thick arrows, resulting in an antisymmetric shear flow v_y at the equatorial plane. The sign of v_y is in agreement with the earlier argument that in general v_y has the same (opposite) sign as the IMF B_y in the northern (southern) hemisphere. As a result, the second term in (5), $B_y \partial v_y / \partial y$, is negative (positive) for positive (negative) IMF B_y . The effect of dawn-dusk asymmetry in the shear flow is thus to limit the growth of B_y and the tilting of the plasma sheet. Note that the dawn-dusk asymmetric flow discussed here is entirely induced by the tilting of the plasma sheet associated with the IMF B_y . There is in fact a regular feature of diverging flows toward the flanks in the central plasma sheet associated with the earthward convection [Huang and Frank, 1994]. The

effect of this diverging flow is to enhance rather than to limit B_y . Nevertheless, this effect is probably important only in the very near-Earth plasma sheet, where the flow becomes azimuthal due to the three-dimensional geometry of the inner magnetosphere [e.g., Zhu, 1993], and relatively less pronounced during the periods of large IMF B_y .

On the other hand, B_y resulting simply from the density compression may be calculated based on

$$(d/dt)(B_y/\rho) = 0 \quad (6)$$

Equation (6) is analogous to the adiabatic energy equation, $d(p\rho^{-\gamma})/dt = 0$, for the plasma pressure p . In particular, the two equations are interchangeable with respect to p and $B_y^2/2\mu_0$, provided that particle motions have only two degrees of freedom, i.e., $\gamma = 2$. This is not a coincidence since (6) is derived under the assumption that v_y is either zero or constant along each field line. For Earth's magnetotail, ρ is in general decreasing down the tail. Equation (6) then implies that a far tail B_y becomes enhanced as plasmas move toward the Earth. To summarize thus far, B_y may be generated by the differential shear motion of a field line and enhanced by the flux tube compression while the effect of dawn-dusk asymmetry in the shear flow, as a result of plasma sheet tilting, is to limit the growth of B_y . Since the last mechanism plays a role only in the presence of a finite B_y , the net result is that plasma sheet B_y will have the same sign as the IMF B_y , consistent with the observations.

3. Plasma Sheet Convection With B_y

In this section we discuss the consequences of plasma sheet convection with a cross-tail magnetic B_y component. We first examine how the presence of a B_y may modify the pressure-balance inconsistency (PBI) argument. For this purpose, we check the consistency between an assumed steady state magnetic field configuration and the condition for adiabatic lossless convection

$$(d/dt)(pV^\gamma) = 0 \quad (7)$$

in the plasma sheet, where V is the volume of a magnetic tube of unit magnetic flux. In the absence of a B_y , Erickson and Wolf [1980] found that the pressure resulting from steady state, adiabatic compression during sunward convection is much higher than the equilibrium pressure required to balance the tail lobe field. This is due to the fact that V near Earth is much smaller than V down tail in standard quiet-time magnetic field models. As shown in the appendix, the flux tube volume of an arbitrary magnetic field line is identical to that of its projection onto a plane, e.g., $y = \text{const}$. Thus, although the addition of B_y increases the field line length, it also increases the total magnetic field strength B such that the flux tube volume remains unchanged. (This has been verified numerically by adding an arbitrary B_y to the noon-midnight plane of the Tsyganenko magnetic field models.) Figure 2 shows the variations of the equatorial normal magnetic field B_{ze} , flux tube volume V , and field line length S down the tail in the noon-midnight plane of Tsyganenko's [1987] model for $Kp = 0$ (hereafter referred to as the T87 model). As indicated, V decreases by a factor of 16 as a flux tube convects from $x = -40$ to $x = -12 R_E$. According to (7), as a flux tube moves from $x = -40$ to $x = -12$, its plasma pressure increases by a factor of 104 for $\gamma = 5/3$. Since this is much larger than the factor of 5 increase in the

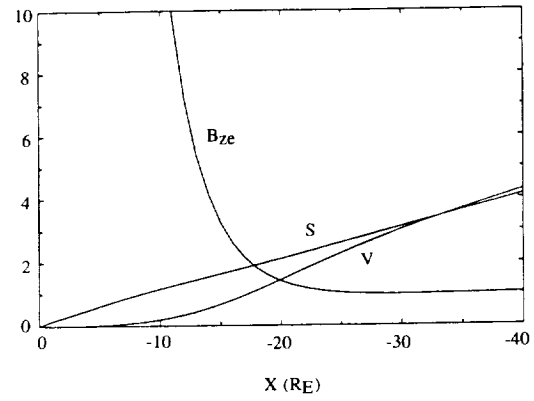


Figure 2. The variations of the equatorial normal magnetic field component B_{ze} , flux tube volume V , and field line length S down the tail in the noon-midnight ($y = 0$) plane of the Tsyganenko [1987] magnetic field model for $Kp = 0$.

corresponding lobe magnetic pressure, it is impossible to maintain the force-balance equilibrium and steady state adiabatic convection at the same time in such a magnetic field configuration. We now add a B_y to the model which, according to (A4) and (7), should not change V nor the increase of plasma pressure due to earthward compression. The same PBI thus persists provided that B_y is small in the tail lobes [Fairfield, 1979]. Thus, although field lines may be lengthened, the addition of B_y does not remove the PBI. Indeed, the PBI with a B_y present seems more severe than in the original argument by Erickson and Wolf [1980]. Moreover, since this result is independent of the mechanism responsible for generating B_y in the plasma sheet, one may conclude that the PBI exists in such magnetic field models, even though the convection process may be accompanied by field line tilting or twisting and the associated field-aligned currents. Note that the fact that a y component of the magnetic field does not invalidate the PBI argument does not imply that B_y is irrelevant since the magnetospheric configuration will evolve differently during convection from the $B_y = 0$ case if a B_y is present.

The implication of the PBI is that standard quiet time magnetic field configurations, such as T87, must change shape in response to plasma sheet convection [Erickson and Wolf, 1980]. We now examine how plasma sheet B_y may evolve during earthward convection. Since quantitative distribution of v_y in the plasma sheet is not available, we consider B_y simply resulting from the density compression term in (4). As for the adiabatic energy equation (7), assuming lossless convection and $\rho \propto V^{-1}$, (6) becomes

$$(d/dt)(B_y V) = 0 \quad (8)$$

Applying (8) to T87 model implies that the equatorial shear magnetic pressure $B_y^2/2\mu_0$ will increase by a factor of 256 as a flux tube moves from $x = -40$ to $x = -12 R_E$, which is even larger than the increase in the plasma pressure (as a result of γ less than 2). The flux tube compression thus leads to a B_y analogue of the pressure-balance inconsistency in the plasma sheet. In this respect, the presence of B_y not only fails to remove the PBI but exaggerates the problem. To estimate the additional lobe flux needed to balance the equatorial total pressure, $p^* = p + B_y^2/2\mu_0$, resulting from adiabatic convection, we carry out the following experiment. Consider an equilibrium magnetic field configuration with $B_y = 0$ and a given

flux tube in the far tail with $p = p_0$ and $V = V_0$. We now add a uniform $B_y = B_{y0}$ to this model which merely adds a constant background pressure and thus does not change the overall equilibrium nor the flux tube volume. Converting the same flux tube in each case to the same near-Earth location results in $p^* = p_0(V_0/V)^\gamma$ and $p_0(V_0/V)^\gamma + B_{y0}^2/2\mu_0(V_0/V)^2$, respectively. After subtracting out the constant magnetic B_y pressure, the difference in lobe flux between the two cases is

$$\Delta B_{\text{lobe}} = B_{y0}[(V_0/V)^2 - 1]^{0.5} \quad (9)$$

Since the B_y correlation may be as high as 50% in the plasma sheet [Lui, 1984], for IMF B_y of the order of $\sim 5 \gamma$ and $V_0(x = -40)/V(x = -12) \sim 16$ as in the T87 model, ΔB_{lobe} may be as large as 40γ . This is much greater than the change of the magnetic field strength in the tail lobes typically observed during the substorm growth phase though an unusually large plasma sheet B_y on association with a very thin current sheet has also been observed prior to the onset of a substorm event [e.g., Sergeev *et al.*, 1993]. As discussed earlier, the dawn-dusk asymmetry of the shear flow will act to reduce the growth of B_y , while the differential shear motion of a field line will lead to further enhancement of B_y . Note that the latter mechanism may play an important role in enhancing B_y of the far tail and perhaps the midtail plasma sheet where the plasma density is less variable. On the other hand, the flux tube compression resulting from earthward convection may be responsible for the strong B_y penetration in the near-Earth plasma sheet. In particular, the result that the penetration efficiency drops from 60% at $x = -20 R_E$ to 30% at $x = -6.6 R_E$ [Nagai, 1987; Sergeev, 1987] may be due to the fact that the convection becomes azimuthal toward the inner magnetosphere.

After showing that the PBI cannot be removed by simply lengthening near-Earth field lines via the addition of B_y , it becomes clear that the only way to achieve steady state convection with $pV^\gamma = \text{const}$ is by inflating the flux tubes via the reduction of the normal magnetic field component there. We now show an example of self-consistent convection models with B_y which is constructed based on quasi-static two-dimensional ($\partial/\partial y = 0$) MHD equations, described by the Grad-Shafranov equation [e.g., Hau and Voigt, 1992]

$$\nabla^2 A + \mu_0 [dp^*(A)/dA] = -\mu_0 j_{\text{dipole}} \quad (10)$$

with the generalized pressure

$$p^*(A) = p(A) + [B_y^2(A)/2\mu_0] \quad (11)$$

where the magnetic vector potential A is constant along a field line. The two-dimensional assumption may appear to be inconsistent with the dawn-dusk asymmetry in association with the B_y discussed above. Observations, however, have indicated that the neutral sheet tilting from the tail axis is usually within a few degrees [e.g., Kaymaz *et al.*, 1994b], we may therefore neglect this effect in the equilibrium model. As before, the functional form $p(A)$ needs to be determined by the adiabatic energy equation (7), while for closure reasons we consider B_y to be entirely due to flux tube compression such that $B_y(A)$ is constrained by (8).

In general, numerical equilibrium solutions consistent also with the thermodynamic constraints (7) and (8) are difficult to construct (for numerical procedure and code description, see the papers by Hau [1991] and Erickson [1992]). This problem, however, is greatly simplified by the fact that for a given xz magnetic field projection, the flux tube volume V is unaffected

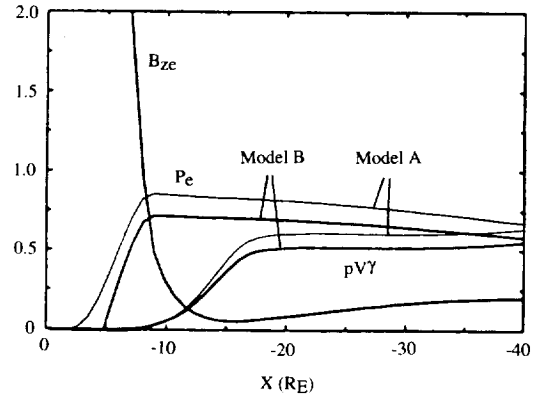


Figure 3. The variations of equatorial B_z , p , and pV^γ down the tail in two-dimensional steady state solutions for $B_y = 0$ (model A) and $B_y \neq 0$ (model B). Both models use the same $p^*(A)$ and thus have the same x - z magnetic field projection and flux tube volumes. The difference between the two pressure curves represents the shear magnetic pressure due to B_y in model B, which is calculated based on (12). To achieve the same $pV^\gamma = \text{const}$ as in model A, a higher p^* with p between the two pressure curves is required in model B. Such a solution will have a smaller B_{ze} than the one shown here.

by the addition of B_y . In this case, previous convection models developed for the case of $B_y = 0$ may be used as a basis for this study. Figure 3 shows the variations of equatorial B_z , p , and pV^γ down the tail in steady state equilibrium solutions for $B_y = 0$ (model A) and $B_y \neq 0$ (model B). Both have the same total pressure $p^*(A)$ but different p and $pV^\gamma = \text{const}$ in the plasma sheet. As indicated, in contrast to Figure 2 for standard magnetic field models, B_{ze} has a minimum in the near tail region. Both models have the same B_x and B_z and thus the same flux tube volumes as a result of (A4). The difference between the two pressure curves represents the shear magnetic pressure due to B_y , which we have calculated based on (8). Since the model has nearly constant V in the plasma sheet, so B_y is nearly constant. (Just as for $pV^\gamma = \text{const}$, the relation $B_y V = \text{const}$ cannot be applied to the inner magnetosphere where particle losses through y drifts may become nonnegligible. For the discussion of this, see Hau [1991, and references therein].) It is clear that model B has smaller p and $pV^\gamma = \text{const}$ owing to the addition of B_y . In order to achieve the same $pV^\gamma = \text{const}$ as in model A, a higher p^* with p between the two pressure curves is thus required which will in turn result in smaller B_{ze} than the one shown here. If instead the far tail plasma pressure is kept the same, the addition of B_y should further increase p^* and pV^γ and yield stronger tail lobe field and smaller B_{ze} . In any case, the result is consistent with our conclusion based on the pressure-balance inconsistency argument.

4. Discussion

The existence of B_y in the plasma sheet is attributed to the field-aligned currents flowing across the equatorial plane and along plasma sheet field lines. For positive (dawn to dusk) B_y , the field-aligned currents flow away from the Earth in the southern hemisphere and toward Earth in the northern hemisphere. For negative (dusk to dawn) B_y , the field-aligned currents flow in the opposite sense. Figure 4 is a sketch of magnetospheric field-aligned currents associated with positive IMF

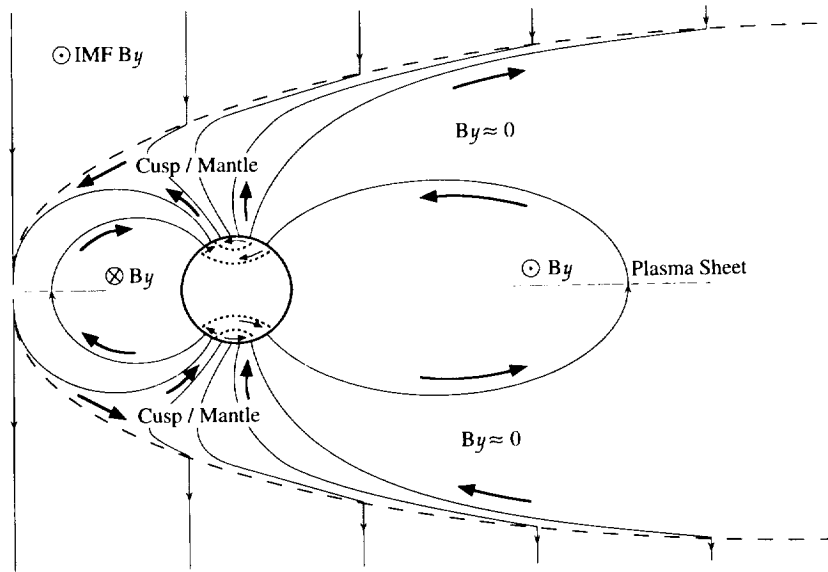


Figure 4. Heavy arrows show the expected closure of currents effecting IMF- B_y penetration into Earth's plasma sheet in the noon-midnight meridional projection for westward IMF B_y . Net current out of the southern ionosphere and into the northern ionosphere along magnetic field lines threading the plasma sheet is required to produce B_y in the central plasma sheet. Net current out of the northern polar cap and into the southern polar cap connecting to the cusp/mantle current system provides for current continuity. The sense of currents is reversed for eastward IMF B_y . The mantle current associated with a rotational discontinuity, together with the interhemispheric currents in the plasma sheet, effectively shield the tail lobes from the IMF B_y .

B_y . The portion of the field-aligned currents in the nightside open field lines is a result of the open magnetotail boundary model developed by *Siscoe and Sanchez* [1987]. In that model the magnetopause is a rotational discontinuity (RD) and the plasma mantle consists of a slow-mode expansion fan. The role of the RD is to insure the MHD coplanarity by rotating the magnetic field vector while the slow-mode expansion fan increases the magnetic field strength and decreases the plasma density. In the presence of an IMF B_y , field-aligned currents must exist in association with the RD in order to shield the slow-mode expansion fan from the B_y . This explains the sense of field-aligned currents on open field lines in Figure 4. The expansion fan model of the plasma mantle has also been used to explain the tilting of the plasma sheet in association with the IMF B_y . As shown by *Siscoe and Sanchez* [1987], the larger the B_y is, the thinner the slow-mode expansion fan is. For purely southward IMF, a B_y may be generated outside the magnetopause (an RD) away from the noon-midnight plane as a result of field line draping. The superposition of a positive IMF B_y on this draping magnetic field then yields a relatively smaller B_y and thicker plasma mantle on the dawnside than on the dusk-side, which in turn causes the tilting of plasma sheet about the tail axis. This explains the sense of rotation of the neutral sheet shown in Figure 1.

On the other hand, the sense of field-aligned currents on the dayside open field line portion is a result suggested by *Pudovkin and Zaitseva* [1993] based on the reconnection model of the magnetopause in the case of large IMF B_y . In particular, these authors suggested that because of the tilt of the magnetopause current from the reconnection line the current perpendicular to the reconnection line will be disrupted in a way such that for positive IMF B_y the current flows away from the Earth in the northern hemisphere and toward Earth in the southern hemisphere. They suggested that these currents may

be closed through the dayside closed field lines or through the polar cap ionosphere to the nightside closed field lines and proposed a global field-aligned current system somewhat similar to that in Figure 4. Their model, however, does not include the field-aligned currents on the nightside open field lines, which as discussed earlier can be explained in terms of the open magnetotail model of *Siscoe and Sanchez* [1987]. These currents can indeed be produced by the same mechanism as that for the dayside field-aligned currents proposed by *Pudovkin and Zaitseva* [1993]. A complete model for the generation of field-aligned currents on association with the IMF B_y is thus emerged. Note that the sense of field-aligned currents on the closed field line portion may also be explained in terms of two-dimensional ($\partial/\partial y = 0$ but $B_y \neq 0$) MHD equilibrium theory. In particular, the field-aligned currents in association with (10) and (11) have the form of [*Voigt and Hilmer*, 1987]

$$j_{\parallel} = \left(\frac{1}{\mu_0} \frac{dB_y(A)}{dA} + \frac{B_y(A)}{B^2} \frac{dp(A)}{dA} \right) \mathbf{B} \quad (12)$$

It is seen that $j_{\parallel}$ vanishes for the case $B_y = 0$. The sense of these currents can easily be illustrated by the linear solution of the form of $p \propto A^2$ and $B_y^2 \propto A^2$ [*Voigt and Hilmer*, 1987], the result being in agreement with the plasma sheet portion of field-aligned currents in Figure 4 based on simple Ampere's law. (It should be remarked that within the two-dimensional static MHD equilibrium theory, the choices of $p(A)$ and $B_y^2(A)$ are entirely arbitrary. The linear solutions not only are mathematically simple but also qualitatively resemble the average, quiet time magnetic field and plasma configurations of the Earth's magnetotail.) The field-aligned currents described by (12) are in contrast to the field-aligned currents produced in general three-dimensional magnetic field models for the case $B_y = 0$ in the plasma sheet, which do not flow across the equatorial plane. Nevertheless, such currents have the physical

meaning for the special case where B_y is present in the plasma sheet as might be expected during the periods of IMF B_y and have indeed been inferred from the observations [Sergeev, 1987]. One might also speculate that the superposition of these field-aligned currents onto a three-dimensional magnetic field model causes a north-south asymmetry in the overall magnetic field configuration due to the asymmetry in the net field-aligned currents.

The linear solutions of (10) and (11) give rise to a B_y decreasing down the tail and a larger B_y in the plasma sheet than in the tail lobes, a result implied by the relation $B_y^2 \propto A^2$ and the fact that $|A|$ is decreasing down the tail and away from the equatorial plane. On the basis of this result, Voigt and Hilmer [1987] argued that the enhancement of B_y in the plasma sheet is a result of MHD equilibrium, rather than of IMF penetration. As mentioned earlier, Moses *et al.* [1985] have explained the relatively high correlation between the neutral sheet B_y and IMF B_y component in terms of the azimuthal tilting of closed field lines. Both arguments are in contrast to the direct IMF B_y penetration interpreted by Lui [1984], which should be stronger, not weaker, on open field lines. The discussion in section 2 and Figure 4 solve this dilemma, which shows how the IMF B_y is shielded less in the plasma sheet than in the tail lobes by the B_y related field-aligned currents. The mantle currents in one hemisphere partially close through the ionosphere, either over the polar cap or along the polar cap boundary to field-aligned currents which flow through the plasma sheet to the other hemisphere and its mantle currents. The field-aligned current flowing between hemispheres is responsible for B_y in the plasma sheet, while field-aligned currents in the plasma sheet and mantle effectively shield the lobe from the IMF B_y .

5. Conclusions

In this paper we have examined the generation and enhancement of plasma sheet magnetic B_y component based on a simple model of Faraday's law and the frozen-in-flux condition and discussed several aspects of plasma sheet convection in the presence of B_y . The results may be summarized as follows.

1. In a convective magnetotail, B_y is generated by differential shear motion of the field line resulting from north-south antisymmetry in the shear flow and enhanced by flux tube compression via earthward convection. Both mechanisms lead to the enhancement of B_y in the plasma sheet in the same direction as the IMF B_y . The growth of B_y , however, may be limited by the dawn-dusk asymmetry in the shear flow velocity as a result of plasma sheet tilting.

2. For a given magnetic field projection on the noon-midnight plane, the addition of B_y yields longer field lines, but it does not affect flux tube volumes. Therefore the conclusion that the pressure balance inconsistency (PBI) exists in standard quiet time, magnetic field models is applied to such a case as well.

3. In the presence of IMF B_y , there exists a B_y analogue of the PBI as a result of earthward compression. The PBI resulting from the shear magnetic pressure is greater than the inconsistency due to the thermal pressure and may further be enhanced by the north-south asymmetry in the shear flow but reduced by the dawn-dusk asymmetry in the shear velocity.

4. For the same pV^γ in the far tail, the presence of B_y in the plasma sheet leads to relatively stronger tail lobe field and smaller equatorial normal magnetic field component B_{z0} .

5. The existence of plasma sheet B_y is associated with field-aligned currents flowing through the equatorial plane. These currents are connected to the IMF B_y related mantle field-aligned currents through the ionosphere and form a current closure which shields the tail lobe from the IMF B_y . A superposition of these currents onto a general three-dimensional magnetic field model shall lead to a north-south asymmetry in the overall configuration.

Appendix

The flux tube volume $V = \int ds/B$ can be written as

$$V = \int \frac{dt}{B_t} \sqrt{\frac{1 + (d\xi_3/dt)^2}{1 + (B_{\xi_3}/B_t)^2}} \quad (A1)$$

where $dt^2 = d\xi_1^2 + d\xi_2^2$ and $B_t^2 = B_{\xi_1}^2 + B_{\xi_2}^2$. By use of the field line definition,

$$\frac{d\xi_1}{B_{\xi_1}} = \frac{d\xi_2}{B_{\xi_2}} = \frac{d\xi_3}{B_{\xi_3}} \quad (A2)$$

and the resulting relation

$$\frac{dt^2}{B_t^2} = \frac{d\xi_1^2 + d\xi_2^2}{B_{\xi_1}^2 + B_{\xi_2}^2} = \frac{d\xi_3^2}{B_{\xi_3}^2} \quad (A3)$$

the flux tube volume is thus simply

$$V = \int dt/B_t \quad (A4)$$

which implies that for a given magnetic field projection, the flux tube volume is independent of the third component of the magnetic field vector. It is also obvious that the tangent vector to the projected magnetic field line, dt , is simply the projection of the original tangent vector at each point.

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- L.-N. Hau, Institute of Space Science, National Central University,
Chung-Li, Taiwan 330.

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